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Equi-Probability Prior to 1650

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Abstract

The assumption that two probabilities can be equal is a conceptual prerequisite for the development of a numerical probability calculus. Such a calculus first emerged in the seventeenth century. Several accounts have been proposed to explain the delayed development of numerical probability, yet it has thus far not been noted that the concept of equi-probability was virtually absent from medieval thought. This article argues that its rise began in the early sixteenth century, a fact that contributes to a better understanding of the preconditions which facilitated the modern mathematization of probability.

Keywords

probability – history of probability – probable opinion – scholasticism – humanism – uncertainty

The exchange of letters between Pascal and Fermat in 1654 marks the birth of numerical probability and modern probability theory. As a matter of fact, the mathematical apparatus of probability theory allows for probabilities to be equal. Equal probability (also known as equi-probability) is also conceivable with respect to older, non-numerical notions of probability, such as those that prevailed in antiquity or the Middle Ages. Yet claims that two probabilities are equal were apparently very rare in the Middle Ages, whereas references to

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comparatively greater or smaller probability abound. This fact is virtually unknown. The few existing works on the history of probability before 1650 pay no attention to the notion of equi-probability, and thus insinuate that the notion was in use alongside others, such as greater or smaller probability. James Franklin's *Science of Conjecture* (2001) quotes two medieval sources, which document that propositions or reasons were sometimes considered equally probable (*aeque probabilis*) in the fourteenth century. However, it has so far not been noted that ascriptions of equi-probability were extremely rare in the Middle Ages – in contrast to the prolific use of the term in early modernity. The notion of equi-probability began being used more widely and – apparently for the first time – systematically in texts on the appropriate choice of opinions. By the end of the sixteenth century, equi-probability had acquired a prominent position in humanist commentaries and translations as well as in scholastic regulations for the choice of opinions.

This fact deserves recognition, considering that modern notions of probability, which represent probabilities as numbers in the zero-to-one interval, conceptually depend on the possibility of regarding the probabilities of different events or propositions as equal. The growing use of equi-probability after 1500 may therefore have been one of the factors that contributed to the probabilistic revolution in the second half of the seventeenth century. The dearth of statements on equal probability in the Middle Ages may, on the other hand, help to explain why it took so long for probability to become quantitative and to figure in mathematical equations. This suggests that an overwhelming majority of medieval scholars did not connect probability with a relation that underlies its mathematization. (This, of course, is not to say that familiarity with the notion of equi-probability was the only or even the most important factor in the multi-causal chain of events that led to the development of the probability calculus). Such considerations warrant a look at the evolvement of equi-probability before 1650, particularly in medieval thought.

Section 1 links the idea of equal probability to medieval concepts of probability. Section 2 discusses the lack of references to equal probability in the Middle Ages. Moreover, it appears that the concept of equi-probability was not mentioned at all in systematic discussions about probable opinions. Section 3 describes the rise of equi-probability in sixteenth-century scholasticism. The parallel rise of equi-probability in early modern humanism is examined in Section 4 to determine whether it was spurred by the Renaissance or by ancient skepticism. Section 5 summarizes the findings and explains in more detail why the introduction of equi-probability might have facilitated the evolution of numerical probability.

1 Equi-Probability and Medieval Concepts of Probability

Probability-related terms, such as 'probable' (*probabilis*), 'truth-like' (*verisimilis*) or believable (*credibilis*), were widely used in the Middle Ages.[1] Their definitions were primarily gleaned from ancient authors, with Aristotle, Cicero, and Boethius as major authorities. The varied uses scholastic and humanistic authors made of probability-related terms are difficult to chart, though the rise of Aristotelianism in the thirteenth century gives some indication of scholastic practices.[2] From the thirteenth century onward, Aristotle's definition of the concept of *endoxon* in *Topics I* became the most important source underpinning the terms *probabilis* and 'probable opinion' (*opinio probabilis*):

> [T]hose opinions are reputable [*endoxa*] which are accepted by everyone or by the majority or by the wise – i.e. by all, or by the majority, or by the most notable and reputable of them.[3]

In line with this definition, 'probable' was generally understood by medieval scholastics to mean 'reputable', 'approved' or 'tenable'. Legal, moral, and intellectual practices were widely guided by probable reasoning and the adoption of probable opinions. The scholastics were well aware – again on the basis of Aristotelian teachings – that the study of human action was beset with uncertainty and had to be rationalized on the basis of 'the probable' rather than on evident knowledge alone.[4] The term 'opinion', which was regularly combined with 'probable', was used by scholastic authors with different albeit related meanings. According to Robert Grosseteste's influential work, 'opinion' (*opinio*) may have three meanings. An 'opinion' could stand for any cognition of a proposition that includes assent. More appropriately, 'opinion' is characterized

1 For the purposes of the present paper, the Middle Ages end in the late fifteenth century. It should not be expected that medieval notions of probability conform to modern understandings of the term.

2 Overviews of medieval notions of probability are found in Thomas Deman, "Probabilis," *Revue des sciences philosophiques et théologiques*, 22 (1933), 260–290; James Franklin, *The Science of Conjecture* (Baltimore, 2001), Ilkka Kantola, *Probability and Moral Uncertainty in Late Medieval and Early Modern Times* (Helsinki, 1994), Rudolf Schuessler, "Probability in Medieval and Renaissance Philosophy," in Edward Zalta, ed., *Stanford Encyclopedia of Philosophy*, <http://plato.stanford.edu/entries/probability-medieval-renaissance/> (December 2014).

3 Aristotle, *Aristotelis libri logicales* (Venice, 1484), 167, i.e., *Topics* 100b20.

4 On probability, see Fn 2; on Aquinas' treatment of uncertain reasoning and opinion, see Edmund Byrne, *Probability and Opinion* (The Hague, 1968). On 'opinion' in the confessional, see Franklin, *Science of Conjecture* and Kantola, *Probability*, 85.

by assent to a proposition combined with a fear of error (that is, an emotionally charged awareness that one's assent might be fallible). Even more appropriately, according to Grosseteste, 'opinion' is defined as a contingent proposition to which a person assents despite fear of committing an error.[5] Grosseteste's second definition of 'opinion' – 'assent to a proposition with fear of error' – was used in the confessional and in doctrines concerning the legitimate choice of opinions.[6] For the present purposes, it suffices to rely on this definition.

Aristotelian endoxical probability can be regarded as a precursor of modern logical or evidential concepts of probability, because it is conditional on external evidence, which Aristotelians considered a guide to truth.[7] A (strong) prima facie tendency toward truth was assumed in particular for statements of the wise (*sapientes*), a term which generally was understood as shorthand for the views of trustworthy experts (*homines probi et docti*) in a given art or science.[8] Although endoxical probability was biased toward external evidence (in contrast to a speaker's own reasons), it is an evidential concept and can be used to confirm propositions, that is, as logical probability.

The *endoxon* as a notion of probability may seem unfamiliar to modern readers, but medieval thinkers were also aware of a precursor to modern frequentist probability. Frequentism claims that probability is a relative frequency of occurrences in a series of events or the mathematical limit of such a relative frequency.[9] I refer to the medieval precursor as 'proto-frequentist', because it differs at closer inspection in significant aspects from modern frequentism. According to proto-frequentism, events that occur 'mostly or for the most part' (*ut frequenter*, *ut in pluribus*) were deemed probable. Aquinas, for instance, stated:

5 Robert Grosseteste, *Commentarius in posteriorum analyticorum*, Vol. 1 (Florence, 1981), lib. 1, cap. 19, 1.

6 See, for instance, Thomas Aquinas, *Summa theologica* (Allen, TX, 1948), I–II, q. 67, a. 3, 874 : "[I]t is essential to opinion that we assent to one of two opposite assertions with fear of the other," and Antonino of Florence, *Summa sacrae theologiae* (Venice, 1582), pars 1, tit. 3, cap. 10, 68: "Opinio autem est acceptio unius partis cum formidine alterius."

7 Modern concepts of probability are discussed, e.g., in Alan Hájek, "Interpretations of Probability," Edward Zalta, ed., *Stanford Encyclopedia of Philosophy*, <http://plato.stanford.edu/entries/probability-interpret/> (December 2011). On Aristotelian *endoxon* as at least a prima facie indicator of truth, see, e.g., Ekaterina Haskins, "Endoxa, Epistemological Optimism, and Aristotle's Rhetorical Project," *Philosophy and Rhetoric*, 37 (2004), 1–20.

8 This is corroborated by the connection between probability, expert statements, and frequent truth in scholastic texts. See, e.g., the quote from Aquinas (see fn 10), and Kantola, *Probability,* 40.

9 See, e.g., Hájek, "Interpretations of Probability," 3.4.

> It is sufficient that you obtain a probable certainty, which means that in most cases (*ut in pluribus*) you are right and only in a few cases (*ut in paucioribus*) are you wrong.[10]

For modern scholars, the proto-frequentist and the endoxical represent the two major medieval notions of probability.[11] For our purposes, it makes sense to discuss whether they lend themselves to the ascription of equal probability.

This does not seem to be the case for proto-frequentism. If 'probable' means 'occurring in most instances', only one of two mutually exclusive events can be probable. Hence, two such events cannot be equally probable.[12] This is a significant restriction, because ascriptions of probability were often used in the Middle Ages to assess mutually exclusive options.[13] If we drop mutual exclusion, it is, of course, conceivable that two compossible events both occur 'for the most part'. Even in that case, medieval writers would not have been in a position to consider them equally probable. This would have required them to compare the relative frequencies of the two events. Yet medieval scholastics – to the best of my knowledge – did not count individual occurrences or calculate relative frequencies. Their understanding of 'for the most part' apparently corresponded to a rough estimate of predominant occurrences, which allowed for a small number of exceptions but was not based on a precise numerical proportion.[14] Medieval authors therefore never came within reach of the numerical form of frequentism that emerged in modern probability theory. Without numerical frequentism, the notion of equal probability could not arise

10 Aquinas, *Summa theologica*, II-II, q. 70, a.2. On Aquinas' use of *ut-frequenter* probability, see Byrne, *Probability and Opinion*, 224; Franklin, *Science of Conjecture*, 124, 203; Kantola, *Probability*, 40. I disagree with the claim (vehemently promoted by Kantola) that Aquinas held a frequentist view of *ut-frequenter* probability in any sense that comes close to the modern understanding of frequentism.

11 See Franklin, *Science of Conjecture*, Kantola, *Probability*.

12 This conclusion remains valid if it is assumed (which I do) that occurrence 'for the most part' only implies (but does not *mean*) probability.

13 This is the case in particular with respect to the assessment of action alternatives in moral theology (see Franklin, *Science of Conjecture*, ch. 4; Kantola, *Probability,* 46; Rudolf Schuessler, *Moral im Zweifel*, Bd. 1 (Paderborn, 2003), and for the choice of opinion in a contrariety of learned opinions, see Thomas Aquinas, *Quaestiones quodlibetales*, ed. Raymund Spiazzi (Rome, 1956), quodl. 3, q. 4, a. 2 [10], 47: "Utrum auditores diversorum magistrorum Theologiae habentium contrarias opiniones, excusentur a peccato, si sequantur falsas opiniones magistrorum suorum."

14 See the examples in Franklin, *Science of Conjecture*, 203 and Schuessler, "Probability," 3.2. I could not find a single medieval example in which a 'ut frequenter' judgment is interpreted as a numerical relative frequency.

from considerations of what occurred 'for the most part'. Hence, medieval proto-frequentist notions of probability actually impeded the emergence of the idea that two events can be equally probable.

The case of endoxical probability differs. Aristotle's concept of *endoxon* renders an ascription of equal probability possible in several ways. Scholastics not only counted but also weighed the opinions of experts. Hence, rival opinions could be equal in terms of number of expert votes and weight, or differ in terms of number and weight, but equal each other in sum.[15] The weight of expert votes for an opinion could also equal that of a multitude of persons without specific expertise. All these options were abundantly used in early modern scholasticism to define and ascribe equi-probability, but apparently no such attempt was made in the Middle Ages. This observation, which at first glance is surprising, can at least partly be explained by the peculiarities of the medieval regulation of moral action under uncertainty. In medieval scholasticism, the word 'doubt' (*dubium* or *dubitatio*) represented different forms of cognitive uncertainty, but with respect to the regulation of moral action (e.g., in the confessional), it had at the latest acquired a specific meaning by the fifteenth century. In their discussions on the decision rule of "In doubt, the safer side has to be chosen" (*In dubio tutior pars est eligenda*), scholars explained that doubt only applied to cases in which an even balance of reasons or uniform ignorance prevailed.[16] On this basis, the practical implications of doubt and probability differed significantly. Probability was a category that legitimized truth-directed acceptance, whereas *dubium*, according to the above stated rule, required moral risk aversion. Risk aversion called for choosing the potentially least sinful alternative, that is, the 'safer side'. In contrast, probability-oriented choice permitted prima facie the adoption of a probable opinion and in contested

15 This option is explicitly mentioned by Konrad Summenhart, *Septipertitum opus de contractibus licitis atque illicitis* (Venice, 1580), q. 100, 562: "Nam ceteris paribus magis adherendum est pluralitati. […] Praeterea si unam opinionem plures teneant, quam aliam: aliam vero graviores, videtur quod si alia sint paria, etiam opiniones illae sint aequalis auctoritatis habendae."

16 For the notion of doubt in medieval thought, see Franklin, *Science of Conjecture*, 67; Kantola, *Probability*. For a more detailed understanding of equally balanced uncertainty and its connection to writings that were particularly important to moral theology, see, e.g., Guillaume d'Auxerre, *Summa aurea in quattuor libros sententiarum* (Frankfurt, 1964), lib. 2, tract. 30, cap. 3, fol. 105, col. 3: "Dubium enim tale est quod habet equales rationes ad hoc quod sit et quod non sit." Angelo de Clavasio, *Summa angelica de casibus conscientiae* (Lyon, 1534), verbum 'opinio', fol. 335: "Dubium vero est motus indifferens in utramque partem contradictionis."."

cases, the choice of a more probable but less safe opinion.[17] It is important to note that probability and safety in this context denote different dimensions of uncertainty. An opinion that is 'more probably' right is less likely to be sinful (at all) than a counter opinion, while 'less safe' implies that an opinion is potentially more sinful, that is, it is a greater sin if it turns out to be a sin at all (which is still uncertain). For confessors, probabilistic choice and choice in doubt thus demarcated two separate domains of moral regulation. This mental separation, reflected in confessors' handbooks, helps explain why medieval scholastics speak of an equal balance of reasons with respect to doubt but not of equal probability.

However, this did not deter medieval authors from ascribing probability to both sides of a question. Hence, a proposition and its negation were both frequently regarded as *probabilis*.[18] This approach could arise directly from the notion of *endoxon*, which ascribes probability to the opinions of the wise. If two rival groups of experts held conflicting opinions, both by definition were thus probable. This possibility, nevertheless, does not imply equal probability of the rival opinions more so than the claim that two persons are both tall implies that they are equally tall. In fact, in scholastic usage, a proposition can remain probable even though a counter-opinion is considered more probable.

Our stock-taking of medieval notions of probability and decision-making under uncertainty thus delivers a mixed picture. On the one hand, obstructions to the use of equi-probability can readily be discovered; on the other hand, no principled reason exists why these obstructions should not have been overcome. It is time, therefore, to look at the actual currency of equi-probability in medieval thought.

17 For the *regula magistralis*, the safety first rule, see Franklin, *Science of Conjecture*, 67. Note that the rule was only applied when in strict doubt, that is, when the agent could not or was not inclined toward an opinion. See Silvester Mazzolini (de Prierio), *Summa summarum quae Sylvestrina dicitur* (Strasbourg, 1518), verbum 'dubium', q. 2, prima: "Sed tamen intellige quod si debet casus quod opinio securior sit minus probabilis notabiliter non est eligenda necessario: quia [...] cessat ratio dubii," or Angelo de Clavasio, *Summa angelica*, verbum 'opinio', fol. 336: "Ergo videtur se exponere periculo qui in universitate opinionum non eligit tutiorem. Quia hoc verum esset quando proprie dubium est: sed quando est opinio secus est: quia nec tunc sumus in dubio: nec consequenter exponit se quis periculo."

18 See Kantola, *Probability*, 29; Schuessler, "Probability," 4.3.

2 Occurrences and Non-Occurrences

As is so often the case, the writings of Aquinas are a good starting point for approaching our subject matter. Aquinas is the only medieval scholastic whose use of probability-related terms has attracted monographic investigation (Byrne, 1968). It is therefore noteworthy that Byrne does not glean any ascription of equal probability to propositions, signs or events from Aquinas. This result is confirmed by searching an Internet database of Aquinas' writings for occurrences of the word stems *probabil** (647 cases) and *verisimil** (106 cases). In no case was the word connected to an ascription of equality (i.e., a variant of *aequalis* or *par*).[19] Aquinas never seems to have ascribed equal probability to propositions or events, although he quite often regarded opinions or events as probable or even more probable than a counter-opinion.

This result is not surprising if the corresponding results in the *Aristoteles Latinus* database are taken into account.[20] A search for *probabil** and *verisimil** yields 184 and 78 cases, respectively. Again, none of these cases is connected to an ascription of equal probability or verisimilitude. Medieval scholastics apparently did not derive the idea of equal probability from the Aristotelian corpus.[21]

The medieval corpus of Aristotle's writings includes Boethius' translation of Aristotle's "Sophistical Refutations," the benchmark translation of the Middle Ages. A crucial occurrence of the Greek *homoios endoxon*, which in the Bekker edition is translated as *aeque probabile* (at 183a1), is phrased as *similiter probabile* by Boethius.[22] Neither Jacob of Venice's nor William of Moerbeke's medieval revisions of Boethius' translation show any divergence in this respect. Hence, medieval readers were not confronted with an Aristotelian reference to equal probability, although one does exist in modern translations of "Sophistical Refutations."

Two of the three ascriptions of equal probability that I unearthed in a sizeable number of medieval sources are mentioned in Franklin's *Science of*

19 The search was conducted in December 2013 in the database of the *corpus thomisticum* (see www.corpusthomisticum.org). I also looked for the writings *equalis*, *eque*, *equi*, etc.

20 The search was conducted in the Brepols *Aristoteles Latinus* database (December 2013), again using the terms *equalis*, etc.

21 This is an indication that the notion of equal probability was not used in antiquity, but I will not delve more deeply into this issue here.

22 Aristotle, *Aristoteles Latinus*, ed. Immanuel Bekker, Vol. 3 (Berlin, 1831), 101; Boethius in *Aristoteles Latinus*, VI. 1–3, 56. There are further passages in Aristotle (*Topics* III, 119b3, 119b15; *Topics* VIII, 161b34) that are translated as "equally probable" or "reputable" today and were translated as *similiter probabilis* in the Middle Ages – see also footnote 43. I will treat the passage from the *Sophistical Refutations* as exemplary in this paper.

Conjecture, who otherwise does not probe into the issue of equal probability. Franklin quotes Hugh of Newcastle (who died in 1322):

> Speaking of the third mode of necessity, Aristotle perhaps and many other philosophers taught that God of necessity produced the world … However, one can argue about this mode of proof with equally or more probable reasons than they have.[23]

Another quote is from Stephen Patrington (around 1380):

> Then I take this proposition as equally probable to the first: God can produce all absolutes [i.e., absolute beings] without any absolute of which it [the first absolute] is not the form nor conversely [i.e., the second being the form of the first].[24]

Finally, a third reference to equal probability can be found in Simon of Faversham's (1260–1306) questions on "Sophistical Refutations":

> An opinion-based *petitio principii* results from the acceptance of a major or minor probable premise, which is equally probable to the conclusion.[25]

These quotes indicate that the notion of equal probability was not entirely absent in the Middle Ages. However, the fact that I only found three examples in a large number of references to probability or greater probability appears significant. It is of course possible that more examples will be discovered if more

23 Hugh of Newcastle, *In Primum Sententiarum*, q. Utrum deus creat aliquid ex se de necessitate, fol. 55v; quoted from K. Michalski, *La philosophie au XIVe siècle* (Frankfurt, 1969), 112: "Loquendo autem de tertio modo necessitatis Aristoteles forte et plures alii philosophi tenuerunt, quod deus de necessitate producit mundum … Tamen circa istum modum ponendi potest argui rationibus aeque probabilibus vel magis sicut sunt rationes eorum." See Franklin, *Science of Conjecture*, 209.

24 Stephen Patrington, Ms. D 28 fol. 1r–3r, St. John's College, Cambridge; quoted from Leonard Kennedy, "Late-Fourteenth-Century Philosophical Scepticism at Oxford," *Vivarium*, 23 (1985), 124–151, 137: "Tunc capio istam propositionem eque probabilem sicud [sic] primam, Omne absolutum potest Deus facere sine omni eo absoluto cuius non est forma nec e contra." See Franklin, *Science of Conjecture*, 209.

25 Simon of Faversham, *Quaestiones super libro elenchorum* (Toronto, 1984), 176: "Petitio principii secundum opinionem est quando procedendo ex probabilibus accipitur maior vel minor quae est aeque probabilis conclusioni." Whether the fact that all three findings are drawn from English scholastics has any significance can only be determined through further investigations.

people start searching for them, and it is also possible that I missed a few occurrences in the texts read. It is nevertheless unlikely that the picture will fundamentally change following further reviews of the texts. Ascriptions of equal probability seem to have been very rare in the Middle Ages.

Konstanty Michalski claimed long ago that scholastic reasoning became increasingly probabilistic (and increasingly skeptical) in the fourteenth century.[26] Michalski also postulated that the scholastic doctrines of probabilism, probabiliorism and equi-probabilism evolved from this trend.[27] All three doctrines are early modern, including their designations, which were coined in the seventeenth century. Michalski seems to suggest that fourteenth-century scholastics already included early forms of probabilism, probabiliorism, and equi-probabilism, but this claim is misleading. The mentioned doctrines are complex systems of moral decision making. Holding a probable proposition to be true does not amount to probabilism, preference for a more probable proposition is less than probabiliorism, and acknowledging that two rival propositions are both probable is not equi-probabilism. Nowhere does Michalski demonstrate that medieval attitudes towards probability or greater probability went beyond such simple basics.

So far, we have mainly discussed scholastic authors. Probability and more generally probability-related terms were of course also used by humanists. In particular, the term *probabilis* was as much a key phrase in humanist rhetoric and dialectic as it was for scholastics.[28] Both sides agreed that dialectic was the art of probable reasoning. However, before the sixteenth century, humanists seem to not have referred more frequently to equal probability than scholastics. I did not come across the phrase 'equally probable' or its cognates in any earlier humanist text – differences between the humanist and the scholastic understanding(s) of probability notwithstanding. Humanists generally shared the endoxical and proto-frequentist interpretations of probability, as outlined above, with scholastics. It can be argued, however, that renowned humanists deviated from this common line in the fifteenth century. This is particularly

26 See Michalski, *Philosophie*.

27 Ibid., 46: "Une analyse minutieuse des textes pourrait prouver que du courant que nous étudions, sont nés, en métaphysique et théodicée, le probabiliorisme, l'équiprobabilisme et le probabilisme, suivant le degré de force persuasive attribuée aux arguments." For the emergence of these terms in seventeenth-century debates on probabilism, see Thomas Deman, "Probabilisme," in Alfred Vacant and Eugéne Mangenot, eds., *Dictionnaire de théologie catholique*, Vol. 13.1 (Paris, 1936), 417–619, 501.

28 See Peter Mack, *Renaissance Argument. Valla and Agricola in the Traditions of Rhetoric and Dialectic* (Leiden, 1993), 31, 146; Marta Spranzi-Zuber, *The Art of Dialectic between Dialogue and Rhetoric* (Amsterdam, 2011), 65.

the case for Lorenzo Valla and Rudolf Agricola, whose understanding of dialectic and probability differed markedly from that of their scholastic contemporaries.[29] For both, probability became a feature of the process of reasoning (i.e., the reasoning itself was called probable), rather than a feature of propositions and the reasoning's premises. However, it is unclear why this alternative should have led to an increased propensity to thinking in terms of equal probability. In fact, there does not seem to be any reference to equal probability in the works of Valla or Agricola. That such references can instead be found in the works of the generation of humanists following Agricola (1444–1485) will be discussed in Section 3.

With regard to equi-probability, it is also important to determine the context in which a probability-related statement occurs. If equal probability had indeed been a theoretically or practically relevant concept in the Middle Ages, we should expect to find references to equal probability in texts on the appropriate choice of opinions. It seems plausible, for instance, that the significance of equal probability would have transpired in discussions of action in doubt or in the regulation of choices between contested opinions of the learned. In fact, equal probability apparently plays no role whatsoever in medieval handbooks of confessors or in texts describing how one ought to cope with the contrariety (*contrarietas*) or variety (*varietas*) of expert opinions. Two of the most widely quoted texts in this respect are *quodlibeta* by Thomas Aquinas and Henry of Ghent.[30] The question is whether expert disagreement must induce doubt in a hearer, and what the hearer has to do when in doubt. We might expect that Thomas and Henry would have touched upon equal probability if this concept had had a relevant function in medieval thought. In fact, however, they did not.

Many late-medieval authors who studied the guidance of conscience followed the lead of Aquinas and Henry of Ghent in debating the problem of a contrariety or variety of opinions among the learned.[31] The influential moral theologians Jean Gerson, Johannes Nider, and Antonino of Florence took a

29 See in particular Rudolph Agricola, *De inventione dialectica libri tres*, critical edition by Lothar Mundt (Tübingen, 1992); Lorenzo Valla, *Repastinatio dialectice et philosophie*, ed. Gianni Zippel, 2 vols. (Padua, 1982); and again Mack, *Renaissance Argument*, 31, 146; Spranzi-Zuber, *Art of Dialectic*, 65.

30 Thomas Aquinas, *Quaestiones quodlibetales*, quodl. 3, q. 4, a. 2; Henry of Ghent, *Quodlibeta* (Paris, 1518), quodl. 4, q. 33, fol. 148.

31 I did not systematically review the treatment of contrariety of opinion by scholastic legists or canonists, among other things, as this would have been a substantial endeavor. However, I have read the relevant passages in Panormitanus' commentary on the *Decretum* (Panormitanus, *Opera omnia,* Frankfurt, 2008). Panormitanus provides a summary of medieval canon law at the end of its truly medieval development. Reference to equal

stance on this issue in the first half of the fifteenth century. As far as I could ascertain, none of them spoke about equal probability. Antonino mentions reasons of equal weight (*rationes aeque ponderantes*) as a basis for doubt, and Nider in this respect refers to equal means (*aequalia media*).[32] Yet this is a familiar characterization of doubt and neither of these authors links it to a discussion of probability.

3 The Rise of Equi-Probability in the Sixteenth Century: Scholasticism

Around the beginning of the sixteenth century, scholastic learning boomed in Paris, Salamanca, and other academic centers. The result, depending on one's perspective, was a continuing or revived flourishing of scholastic thought in the early modern period.[33] A dynamically evolving discourse on probability was one of the key features of scholasticism in early modernity, leading, among other things, to a recognition of equal probability. In fact, equal probability got an early start. The ground for its systematic use was prepared by such luminaries of sixteenth-century scholasticism as Konrad Summenhart and John Major, although none of them took the final step toward equi-probability.

Konrad Summenhart, a German who taught in Tübingen and acted as an economic advisor to the Fuggers, was one of the founders of the influential Tübingen School of scholasticism in the late fifteenth century.[34] Question 100 of Summenhart's epic work on contracts (*Septipertitum opus de contractibus*) contains an extensive discussion on the contrariety of opinions among doctors and the appropriate choice of opinion in this context. Summenhart lists seven criteria for adopting an opinion over another in cases of disagreement among the learned. This list became a blueprint for subsequent sets of criteria for the

probability may have been expected here if it had been at all relevant in medieval legal tradition, but none is made.

32 See Antonino of Florence, *Summa theologiae*, 69; Johannes Nider, *Consolatorium timoratae conscientiae* (Venice, 1532) 50v.

33 The question whether a comparative revival of scholasticism occurred around 1500 cannot be discussed here. It implies, of course, a fifteenth-century dip in scholasticism (but by no means a full breakdown). The boom of Iberian scholasticism in the sixteenth century and events at the University of Paris seem to confirm a cyclical upturn in the early sixteenth century, see Ricardo Garcia-Villoslada, *La universidad de Paris durante los estudios de Franciso Vitoria OP* (Rome, 1938).

34 The other founder was Gabriel Biel. On Summenhart, see Jussi Varkemaa, *Conrad Summenhart's Theory of Individual Rights* (Leiden, 2012).

rational adoption of the opinions of others, which can often be found in scholastic treatises on conscience. For our purposes, it suffices to focus on the treatment of equality in Summenhart's list. Criterion five for preferring one opinion over another specifies that more doctors *ceteris paribus* adhere to the preferred opinion. Under this heading, Summenhart presents a case in which more doctors adhere to one opinion and "weightier" (*graviores*) doctors to another. In this case, the two opinions have equal authority (*auctoritas aequalis*).[35] This is a step toward the ascription of equal probability, because such considerations occur in the context of a choice of opinion rather than in the context of moral action in doubt. However, Summenhart does not yet use the language of probability when acknowledging equal authority.

That this connection was not silently implied can be gleaned from another list of seven criteria. Summenhart offers seven cases (obviously, there is something special about the number seven in a work called *Seven-part Work on Contracts*), in which an agent mortally sins by choosing a contested opinion. In the fifth case, the agent's conscience indicates that all available opinions imply a mortal sin, so that the agent 'equally believes' (*aeque credit*) in the sinfulness of the respective opinions. In other words, the agent is in doubt about the right choice of action while all options appear equally sinful. Notably, Summenhart does not use the language of probability here, but uses it in the fourth case, in which the agent has a 'probable conscience' (*conscientia probabilis*) that an opinion implies a mortal sin. He explains that the conscience (i.e., the judgment of conscience) is called 'probable' because it leans more toward the belief that an opinion is mortally sinful than the opposite.[36] Hence, an equilibrated conscience is by definition still not probable for Summenhart.

John Major taught philosophy and theology between 1496 and 1518 in Paris to such diverse students as Erasmus of Rotterdam and Francisco de Vitoria.[37] His nominalist doctrines had a strong influence on the otherwise Thomist School of Salamanca. Major discussed the choice of opinions at length in the prologue to his commentary on the fourth book of Lombard's Sentences. Question 2 deals with conflicting opinions in moral matters. The possibility of equally strong or weighty considerations on both sides is addressed several

35 Summenhart, *Opus de contractibus,* q. 100, 562: "Praeterea si unam opinionem plures teneant, quam aliam: aliam vero graviores, videtur quod si alia sint paria, etiam opiniones illae sint aequalis auctoritatis habendae."

36 Summenhart, *Opus de contractibus*, q. 100, 560: "agens habet probabilem conscientiam, quod sit mortalis & dicitur probabilis: ubi plus declinat ad credendum quod sit mortalis, quam quod non sit talis."

37 On Major (also written Mair), see Alexander Broadie, *The Circle of John Mair* (Oxford, 1985).

times. Major writes that one side equals the other in reasons (*ratio … aequalet*), or that assertions are equally apparent (*aeque apparentes*), or that the motives are equal for both sides (*aequalia motiva*), but he never speaks of equal probability.[38] Again, the import of these claims of equality for our discussion is that they occur in the context of the choice of contested opinions.

The first mention of equal probability in a systematic discussion of the choice of opinions was, to the best of my knowledge, made in Silvestro Mazzolini's *Summa summarum* (1515), a highly influential confessor's handbook. Under the heading 'doubt' (*dubium*), Mazzolini writes:

> Doubt is twofold. For instance, probable if the probable reasons for both sides are more or less equal, and scrupulous if someone out of a slight suspicion fears that somewhere lurks a sin.[39]

The main point is that Mazzolini does not only refer to equally strong reasons but to (roughly) probable reasons that are equal – and in case this is not a clear enough example of equal probability, he later adds: "Yet if probability is equal on both sides, the safer part is to be chosen."[40]

Distinctions between different kinds of doubt were common in the Middle Ages, often recalling Aquinas' distinction between truly balanced doubt that excludes assent and doubt that merely motivates a fear of error without precluding assent.[41] The notion of probable doubt also appeared in Aquinas and other medieval scholastics, but it merely indicated a well-motivated doubt.[42] It is noteworthy that Mazzolini invests the concept of probable doubt with a different meaning, one that refers to a balance of probable reasons. He thus builds on a tradition that characterized doubt through an equilibrium

38 John Major, *In quartum Sententiarum quaestiones* (Paris, 1516), fol. 3: "Secundo modo recipitur aliqua doctrina sic probabilis ut non liceat ei contraire: sed danda est ei tamen expositio per alia scripta consimilis auctoritatis vel maioris per rationem naturalem quia ratio canoni aequalet." (fol 4): "Sic enim assertiones eorum oppositae sint aeque apparentes." (fol. 5): "Non potest hoc assentire quod una pars est faciendum cum est inter aequalia motiva: sed quicumque contravenit conscientiae suae peccat."

39 Mazzolini, *Summa summarum*, verbum 'dubium': "Dubium est duplex. Scilicet probabile: cum rationes probabiles ad utramque partem sunt quasi aequales: et scrupulosum: quando quis ex levi suspitione timet alicubi esse peccatum."

40 Ibid., "Si tamen probabilitas hicinde esset aequalis tutior pars eligenda est."

41 See Aquinas, *Quaestiones quodlibetales*, quodl. 8, a. 13.

42 See Aquinas, *Summa theologica,* II–II, q. 189, a. 8, 2007; Adrian of Utrecht, *In quartum Sententiarum* (Paris, 1516), fol. 71, col. 4. *Dubium probabile* could also simply refer to an open debated question, see Thomas Woelki, *Lodovico Pontano* (Leiden, 2011), 196.

of reasons, but additionally emphasizes the role of probability on both sides. This, as documented, leads him to an explicit ascription of equal probability. It is also noteworthy that no other major confessor's handbook from the period mentions equal probability. Alphabetically ordered handbooks, such as the "Angelica," "Pisana," "Rosella," or "Tabiena" had wide currency and probably did much to spread interest in matters of choosing an opinion.[43] However, they contain no reference to equal probability under the relevant headings of 'doubt' or 'opinion'.

Mazzolini's conception was picked up in a very rare and even at the time not widely known treatise by the Spanish Hieronymite monk Barnabas de Rosalibus. Barnabas published his extensive treatise on penitence and the variety of opinions in 1540.[44] He quotes Aquinas, but hardly any other medieval or contemporary authority. His moral outlook leans more toward the austere side, although he claims to take a middle course between Scylla and Charybdis on the thorny question of choosing an opinion.[45] In the beginning of his analysis, Barnabas introduces the distinction between an equal and an unequal probability of rival opinions, a distinction that he subsequently applies in the discussion of cases. Barnabas contends that the opinions of the learned (*doctorum opiniones*) can be equally probable (*aeque probabiles*) or unequally probable to our intellect or the judgement of their examiners. In the case of equal probability, the opinions are confirmed by an equal weight of reasons or authorities.[46] Later in the treatise, he returns to this definition several times and discusses choices between equally probable opinions.[47]

Barnabas' approach might have remained uninfluential had it not been taken up by the much better known and more frequently quoted Antonio de Cordoba. Cordoba is one of the major representatives of sixteenth-century Spanish scholasticism. He deals with the choice of contested opinions in question 3 of the second book of his "Quaestionarium theologicum" from 1569. Cordoba explicitly gives credit to Barnabas when he distinguishes three options of choice between contested opinions. The third option is a choice between equally

43 See Giovanni Cagnazzo, *Summa Summarum, quae Tabiena dicitur* (Bologna, 1520), Angelo Clavasio, *Summa Angelica*, Baptista de Salis, *Summa Rosella* (Venice, 1495), Nicolaus de Auximo, *Supplementum Summae Pisanellae* (Nuremberg, 1488).

44 See Barnabas de Rosalibus, *Relectio de tribus poenitentiae partibus atque opinionum varietate, quae videlicet tenenda sit* (Valencia, 1540). Fols. 81v to 111v are *de varietate opinionum*.

45 Ibid., 82r.

46 Ibid., 83r: "Doctorum opiniones, […], possunt esse apud intellectum nostrum vel eas examinantium iudicium non aeque probabiles, hoc est non equalis ponderis rationibus vel autoritatibus innixae, aut aeque probabiles videlicet aequalibus mediis probatae."

47 See ibid., 97r, 97v, 103r.

probable opinions, and in this case, Cordoba (like Mazzolini) invokes the main medieval rule of moral choice in doubt, according to which the safer side must be chosen.[48]

Roughly one decade later, the Salamancan professor Bartolomé de Medina invented the doctrine that came to be called 'scholastic probabilism'.[49] Medina's innovation spread like wildfire among Catholic moral theologians and became the most important driver of the rich scholastic probability discourse of the seventeenth century. Medina distinguishes between the right, erroneous, doubting, and scrupulous conscience (*c. recta*, *erronea*, *dubia*, *scrupulosa*).[50] Sticking to tradition, he does not mention equal probability when discussing problems of doubt. Medina instead speaks of equal doubt (*aequale dubium*) or equal risk (*aequale periculum*).[51] Only then does he turn to the question concerning the conditions under which one opinion from a set of diverse opinions (*diversae opiniones*) may be adopted. It is here that Medina defines opinion and probable opinion and refers back to Conradus (i.e., Summenhart) as an authority on the choice of opinions. In his second conclusion, he states that one may (*ceteris paribus*) adopt any of two equally probable opinions.[52]

With Medina and the unfolding of scholastic probabilism, the concept of equi-probability became a standard element of analysis and regulation of choice of opinion and moral action in early-modern scholasticism. It is not exaggerated to say that almost all Catholic moral theologians of the seventeenth century took a stance on Medina's probabilism in one way or other.[53]

48 Antonio Cordoba, *Quaestionarium theologicum* (Venice, 1604), 12: "Tertia propositio. Quando opiniones sunt vel creduntur aeque probabiles semper id quod videtur minus malum et tutius tenendum est, quando est dubium de peccato mortali."

49 See Bartolomé de Medina, *Expositio in primam secundae Angelici Doctoris D. Thomae Aquinatis* (Venice, 1580), q. 19, a. 6. Probabilism allows adherence to a less probable opinion even though the counter-opinion may be more probable. This doctrine was bitterly attacked by Blaise Pascal in the middle of the seventeenth century after it had become mainstream among Catholic theologians. On scholastic probabilism, see T. Deman, "Probabilisme," 417–619; Franklin, *Science of Conjecture*; Rudolf Schüssler, *Moral im Zweifel*, Bd. 2 (Paderborn, 2006).

50 A 'scrupulous conscience' is one that is beset by scruples. 'Scruples' (*scrupuli*), in turn, were irrational or unmotivated fears in the terminology of the confessional, see Franklin, *Science of Conjecture*, 71; Sven Grosse, *Heilsungewissheit und Scrupulositas im späten Mittelalter* (Tübingen, 1994).

51 Medina, *Expositio*, 177.

52 Ibid., 178: "Secunda conclusio. Quando utraque opinio tam propria quam opposita est aeque probabilis, licitum est indifferenter utramque sequi."

53 For the seventeenth century, see references to equi-probabilism in Sven Knebel, *Wille, Würfel und Wahrscheinlichkeit* (Hamburg, 2000).

Hence, I do not consider it necessary to track the development of equi-probability beyond Medina. This concept can subsequently be found in virtually all probabilist and anti-probabilist writings.

4 The Rise of Equi-Probability in the Sixteenth Century: Humanism

We have so far looked at scholasticism in the sixteenth century. That century was, of course, also a momentous period for early modern humanism. In dialectic, Agricola's innovations spread across Europe. Melanchton and other Protestant humanists developed a keen interest in rhetoric and dialectic. Last, and for our purposes not least, the notion of equi-probability can increasingly be found in humanist writings. Agostino Nifo's commentary on "Sophistical Refutations," which was published in 1534, retains Boethius' *similiter probabilis* in the translation of the text, but refers to 'equal probability' in the commentary.[54] Around the same time, Simon Gruener used the term *aeque probabilis* in his commentary on Aristotle's *Topics*.[55] The rising familiarity of equal probability was finally also reflected in translations of Aristotelian texts, such as "Sophistical Refutations" (see 183a). The old Boethian rendering of *homoios endoxon* as *similiter probabile* remained widespread throughout the sixteenth century. Yet some translators began to use *aeque probabile* instead of *similiter probabile*, as evinced in Nicolas de Grouchy's translation of "Sophistical Refutations" from 1550.[56] This translation was integrated into the 1593 edition of Aristotle's *Works* by Joachim Perion – thus preparing the way for the Bekker edition quoted above.[57]

It is difficult to say whether humanist usage of equi-probability preceded scholastic usage or vice versa. Apparently, both intellectual traditions increasingly began to use the notion of equal probability in the decades following

54 Agostino Nifo, *Expositiones in libros de sophisticis elenchis Aristotelis* (Venice, 1542), 70.

55 Simon Gruener (Grynaeus), *In librum octavum Topicorum Aristotelis commentaria* (Basle, 1556), 92.

56 Nicolas de Grouchy, *De reprehensionibus sophistarum liber unus* (Cologne, 1561), 90. This translation first appeared 1550 in Paris.

57 Joachim Perion, *Aristotelis operum, quotquot extant, Latina editio* (Frankfurt, 1593), Vol. 1.1, 470. Joachim Perion, *Aristotelis Topicorum libri octo* (Cologne, 1561), 71, 249 also translates *Topics* III, 119b3 and *Topics* VIII, 161b34 as *aeque probabilis*. Both passages were before translated as *similiter probabilis*, see Agostino Nifo, *Aristotelis Topicorum libri octo* (Venice, 1569), 112, 264; Aristotle, *Aristotelis libri logicales* (Venice, 1484), 115*, 150*. [*The book is not paginated, therefore I quote the pdf page of the downloadable version (Bavarian State Library)].

1500.[58] However, its spread does not seem to have been the result of renewed interest in translations from Greek. The earliest uses in the sixteenth century I found were in commentaries or works on moral theology. Only later were translations of the Aristotelian corpus adapted. Hence, they seem to reflect a new option for translators which resulted from the availability of the notion of equal probability.

Did the Renaissance of ancient skepticism have an impact on these developments? This is an obvious question to ask, because Pyrrhonian skepticism creates doubt by emphasizing the equality of reasons on both sides of a dispute. An equality of reasons can easily be related to equal probability in Renaissance thought. Richard Popkin's (and subsequent) work on the recovery of ancient, and above all, Pyrrhonian skeptical texts and doctrines in early modernity documents how important the return of writings such as those of Sextus Empiricus were for the rise of modern philosophy.[59] In response to new research, Popkin shifted the onset of renewed interest in Pyrrhonism back in time from Erasmus to Savonarola. However, it has been shown that the starting point should best be relocated to the early fifteenth century, when Ambrogio Traversari in Florence produced a Latin translation of the book on Pyrrho in Diogenes Laertius' *Lives of Eminent Philosophers*.[60]

Against this background, the question may be asked as to whether the notion of equi-probability or phrases like *aeque probabilis* already appeared in skeptical writings or translations thereof *before* they began spreading in the early decades of the sixteenth century. I have probed this question, but a complete review of the literature exceeded my powers. Fortunately, documented interest in ancient Pyrrhonism only began to rise gradually before the second half of the sixteenth century. Gianfrancesco Pico della Mirandola's *Examen*

58 For uses of probability in rhetoric, logic, law, and medicine mainly in the early modern era, see Ian Maclean, *Interpretation and Meaning in the Renaissance* (Cambridge, 1992); Ian Maclean, *Logic, Signs, and Nature in the Renaissance* (Cambridge, 2002). Yet to the best of my knowledge, Maclean's work contains no directly relevant material for the present discussion of equi-probability. Maclean *Logic*, 181, shows that some Renaissance writings on medicine contain quantifications of predictive success (e.g., an error of less than one in thousand, or being right in half of the cases), but these are apparently not used to form equations or to formulate a concept of equal probability. Nevertheless, medical or Galenist considerations of probability may have played a role in establishing an early-modern frequency interpretation of probability.

59 Richard Popkin, *The History of Scepticism from Savonarola to Bayle* (Oxford, 2003).

60 Luciano Floridi, *Sextus Empiricus. The Transmission and Recovery of Pyrrhonism* (Oxford, 2002).

vanitatis (1520) appeared after Mazzolini's first printed reference to equi-probability. The most valuable place to look for occurrences of *aeque probabilis* (or cognates) is therefore Traversari's own translation. Interestingly, the modern Loeb translation of Diogenes Laertius' book on Pyrrho speaks in one passage of an equal probability on both sides.[61] But Traversari does not express the respective sentence in the language of probability. Instead, he mentions equal persuasiveness (*persuasiones aequales*).[62] This is not because Traversari shunned the notion of probability in general or in the context of skeptical thought, as little later in the same text, he speaks about *probabilia*.

It is still possible, of course, to assume that the skeptical preoccupation with equal reasons on all sides facilitated the spread of the notion of equal probability. Such an assumption makes perfect sense if we focus on the wider context of an equality of reasons or evidence rather than on occurrences of a key phrase. However, renewed interest in ancient skepticism does not play a singular role in this broader context. As indicated above, the scholastic concept of doubt, in particular in its technical meaning in moral theology, already implied an equal balance of reasons or evidence. The rise of early modern moral theology is therefore as probable a background for the increased interest in equal probability as the Renaissance of ancient skepticism. This is not to say that further research could not uncover a specific role of skepticism. It just means that at the present stage of our knowledge, no noteworthy contribution of skeptical Renaissance can be ascertained.

5 Conclusion

The concept of equi-probability seems to have become an object of systematic use and conscious reflection after the sixteenth century. Discussions concerning the choice of opinions in cases in which the opinions of the learned differ drove this development in scholasticism. Earlier discussions of this problem and considerations of uncertain moral agency lack references to equal

61 Diogenes Laertius, *Lives of Eminent Philosophers* (London, 1925), 9. 79, 490: "They showed then, on the basis of that which is contrary to what induces belief, that the probabilities on both sides are equal."

62 Diogenes Laertius, *Vitae et sententiae philosophorum*, trans. Ambrogio Traversari (Venice, 1475), 327*: "Demonstrabant itaque ex his quae contraria sunt persuasiones aequales esse persuadentibus." [*The book is not paginated, therefore I quote the pdf page of the downloadable version (Bavarian State Library)]. The same sentence is still translated as *persuasiones aequales* in the 1692 Casaubon edition.

probability, but draw either on the concept of doubt or on a strict ordering (>, <) of more probable and less probable opinions. We have no compelling reason to think that this state of affairs could not have prevailed longer than it did. It seems that the greater detail of early modern treatments of disagreement among the learned (see Summenhart and Major) increased the salience of the concept of equal probability and thus stimulated its use. Prolific disseminators, such as Mazzolini and Cordoba, spread the concept until Bartolomé de Medina included it in his path-breaking exposition of scholastic probabilism. Subsequently, the concept of equi-probability became endemic in the very rich scholastic discourse on probability in the seventeenth century.

Humanists apparently also began using the notion of equal probability around 1500. Agostino Nifo and Simon Gruener used the term, and translations of Aristotle's works soon followed suit. The notion of equal probability thus proliferated on a broad front in early-modern European thought.

It seems plausible that the triple (>, <, =) of ordering relations between probabilities facilitated the emergence of numerical representations of probability, because these representations presuppose the possibility of such an ordering. However, our case appears a bit more complex when examined more closely. The first numerical representations of probability pertain to games of chance and were not couched in the language of probability, but of chances or possibilities. Neither the Pascal-Fermat letters on a mathematical solution to the so-called 'problem of points' (in games of chance) nor Huygens' treatise on games of chance connect the discussed issues with the concept of probability.[63] There apparently was no need for the concept of equal probability as long as the notion of equal chance or equal possibility was available. Nevertheless, Jacob Bernoulli's *Ars conjectandi* (1713) soon connected chance and probability, and this step in the evolution of modern probability theory presupposed the

63 See Lorraine Daston, *Classical Probability in the Enlightenment* (Princeton, 1988), Anders Hald, *A History of Probability and Statistics and Their Applications before 1750* (New York, 2003), Christiaan Huygens, "De ratiociniis in ludo aleae," in Frans Schooten, ed., *Exercitationes mathematicae* (Leiden, 1657), 517–534. Even Juan Caramuel, apparently the first scholastic author who used mathematical probability (and Huygens' treatise) in his *Mathesis nova*. did not systematically connect games of chance and probability in this work. However, he attributes probability and verisimilitude to chance events in passing, see Juan Caramuel y Lobokowitz, *Mathesis bíceps* (Campania, 1670), 1033. Caramuel's probabilism is analyzed in Julia Fleming, *Defending Probabilism. The Moral Theology of Juan Caramuel* (Washington DC, 2006).

possibility of a weak ordering of probability with the triple (>, <, =).[64] The connection of chance and probability in Bernoulli was foreshadowed by Leibniz who as early as the 1670s sought a mathematical treatment of degrees of probability.[65] It is plausible that the wide availability of the concept of equiprobability from the late sixteenth century onward facilitated these developments.

64 See Jacob Bernoulli, *Ars conjectandi* (Basle, 1713), 211 and in particular part four of the book. Abraham de Moivre, *The Doctrine of Chances* (London, 1718) also connects probability and chance.

65 See Ivo Schneider, "Leibniz on the Probable," in Joseph Dauben, ed., *Mathematical Perspectives* (New York, 1981), 201–219, esp. 204.